BUILDINGS SUSCEPTIBLE TO TORSIONAL-TRANSLATIONAL COUPLING

O.A. Pekau and H.A. Gordon

SYNOPSIS

Coupling between the torsional and translational response is introduced in buildings when the centre of mass and centre of resistance are not coincident. The coupling is amplified when the natural frequencies in torsion and translation are close. In this paper an attempt is made to provide general guidelines whereby situations for which lateral-torsional amplification that can lead to potentially severe earthquake response may be identified. The effects of different building plan configurations and arrangements of lateral load-resisting elements are first examined for idealized structures. The results show that buildings having uniformly distributed lateral resistance are especially susceptible to coupling, regardless of the plan configuration. Next, a multi-storey frame building is studied for earthquake excitation, with special regard for the effect of closeness of frequencies in the presence of small eccentricity. Here, it is concluded that the maximum seismic response of stiff buildings is more sensitive to coupling than that of corresponding flexible buildings, and also that occurrence of lateral-torsional coupling leads to a decrease in total base shear.

RESUME

L'interaction entre la réponse de translation et la torsion a lieu lorsque le centre de gravité et le centre de rigidité ne coincident pas. Cette interaction est amplifiée lorsque les fréquences naturelles de torsion et latérales sont rapprochées. Cette communication présente les moyens nécessaires afin d'identifier ces situations risquées lors des tremblements de terre sévères. Les bâtiments avec rigidité latérale sont particulièrement susceptibles à l'interaction. Par contre, les cadres multi-étagés sont étudiés en présence d'une excentricité pour voir l'effet des fréquences trop rapprochées. Il est conclu dans cette étude que la réponse maximale d'un bâtiment rigide est plus sensible à l'interaction de l'effet latéral avec celui de torsion que dans le cas des bâtiments flexibles.

Oscar A. Pekau received the M.A. Sc. degree from the University of Toronto in 1964, the M. Sc. (Eng) from the University of London in 1965 and the Ph.D. from the University of Waterloo in 1971. At present he is Associate Professor in the Department of Civil Engineering at Concordia University.

Henry A. Gordon received his B. Eng. in 1972 and his M. Eng. in 1977, both from Concordia University. Following a number of years with the Aluminum Company of Canada and Canadian Copper Refineries Ltd. he joined Noranda Mines Ltd., where he is currently Project Engineer.

INTRODUCTION

Observations following recent earthquakes (1) have revealed structural damage attributed to unexpected torsional motion of buildings. In less severe circumstances, wind-induced torsional-translational oscillations have been identified as the cause of considerable distress to modern multi-storey buildings. Thus, the question of coupled lateral-torsional motions of structures during earthquake excitation warrants careful consideration and various investigations dealing with this problem have been reported in recent years (2,3,4).

Newmark (5) developed a simple estimate for the equivalent eccentricity between the centres of stiffness and mass for symmetrical structures to account for the torsional component of ground motion in a simple manner. The latter results from the spatial derivatives of the two horizontal components of ground translational acceleration which produces non-uniform excitation over the base of the structure. Tso (6) and Tso and Asmis (7) demonstrated the danger of coupling between torsional and translational oscillation in nominally symmetrical structures caused by yielding during purely translational excitation.

Hoerner (8) examined the dynamic characteristics and seismic response of a class of tall buildings modelled by continuous cantilevers. The investigation determined that coupling can lead to increases in maximum response by as much as 40 - 90% over that of the corresponding uncoupled structures, and consequently recommends designs which avoid coupled behaviour wherever possible. In the recent work of Kan and Chopra (9,10) lumped parameter models for both single and multi-storey buildings were considered. Behaviour of the simple model (9) revealed the important conclusion that coupling always reduces the maximum base shear, whereas for the multi-storey structures (10) a method of analysis was proposed whereby an asymmetrical 3N degree of freedom system is approximated with good accuracy by considering the mode shapes of the corresponding uncoupled structure of N storeys. In both of the above investigations lateral-torsional coupling was shown to depend not only on the eccentricity but also on the closeness of the translational and torsional frequencies, with the result that peak stresses in the usually critical peripheral elements of an asymmetrical structure are expected to be a maximum when the natural frequencies are not well separated.

Commentary K of the National Building Code of Canada (12) makes special reference to the problem posed by buildings when the separation of the fundamental lateral and the fundamental torsional frequencies are not separated by at least ±20%. However, no guidelines are at present available to designers for identifying such cases a priori, without first performing a three-dimensional dynamic analysis to determine the dynamic properties of the building. Because of the computational effort required to deal with the increased number of degrees of freedom involved in the requisite three-dimensional analysis of torsionally coupled multi-storey structures, approximate methods become desirable. One such method for wall-frame structures has recently been proposed by Rutenberg, Tso and Heidebrecht (11).

In this paper a simplified structural model for buildings is first examined and a parameter to measure its expected degree of modal coupling is discussed. It is obvious from the preceding studies that this parameter should depend on both the eccentricity between centres of stiffness and mass as well as on the separation of translational and rotational frequencies. From the viewpoint of structural planning and design, different geometries of building plan configurations as well as different strategies for the arrangement of the lateral load-resisting elements are then studied. Cases considered include rectangular, L and T-shaped plan configurations. Finally, in order to examine the importance of the degree of modal coupling on maximum seismic response, a 4-storey frame building exhibiting frequency separations covering a range of values is investigated for unidirectional earthquake excitation.

COUPLING BASED ON SIMPLE MODELS

Degree of Coupling

The single mass shown in Fig. 1 is a structural representation of a single-storey building with its mass lumped at the roof level. The model has asymmetrical stiffness and three degrees of freedom, translation in the x and y directions and rotation Θ.

The centre of rigidity is defined by the relationships

$$(k_{x_1} + k_{x_2})\ (b+e_y) = (k_{x_3} + k_{x_4})\ (b-e_y) \qquad (1)$$

$$(k_{y_1} + k_{y_4})\ (a+e_y) = (k_{y_2} + k_{y_3})\ (a-e_x) \qquad (2)$$

where e_x and e_y represent eccentricities in the x and y directions, respectively, and k_{xi} and k_{yi} are corresponding stiffnesses of the i^{th} column. The equations of motion for free vibration, considered about the centre of mass, may be written in the form

$$M \begin{Bmatrix} \ddot{x} \\ r\ddot{\theta} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} K_x & -\frac{e_y}{r} K_x & 0 \\ -\frac{e_y}{r} K_x & K_\theta & \frac{e_x}{r} K_y \\ 0 & \frac{e_x}{r} K_y & K_y \end{bmatrix} \begin{Bmatrix} x \\ r\theta \\ y \end{Bmatrix} = \{0\} \tag{3}$$

where

$$K_x = \Sigma k_{xi} = k_{x1} + k_{x2} + k_{x3} + k_{x4} \tag{4}$$

$$K_y = \Sigma k_{yi} = k_{y1} + k_{y2} + k_{y3} + k_{y4} \tag{5}$$

$$K_\theta = \frac{1}{r^2} \Sigma (x_i^2 k_{yi} + y_i^2 k_{xi}) = \frac{1}{r^2} (a^2 K_y + b^2 K_x) \tag{6}$$

and M = total mass; r = polar radius of gyration.

To reduce parameters, the problem considered herein will be simplified by assuming one-fold symmetry about the y axis; thus, $e_x = 0$ in what follows. For harmonic motion, the eigenvalue problem is described by the determinantal equation

$$\begin{vmatrix} K_x - \omega_i^2 M & -K_x \delta & 0 \\ -K_x \delta & K_\theta - \omega_i^2 M & 0 \\ 0 & 0 & K_y - \omega_i^2 M \end{vmatrix} = 0 \tag{7}$$

where

$$\delta = \frac{e_y}{r} \tag{8}$$

and ω_i = natural frequency of the i^{th} mode. Expanding and solving leads to the expression for the natural frequencies

$$\omega_1^2 = \tfrac{1}{2} \left\{ \frac{K_\theta}{M} + \frac{K_x}{M} - \left[\left(\frac{K_\theta}{M} - \frac{K_x}{M} \right)^2 + 4 \frac{K_x^{\,2}}{M^2} \delta^2 \right]^{\frac{1}{2}} \right\} \tag{9}$$

$$\omega_2^2 = \tfrac{1}{2} \left\{ \frac{K_\theta}{M} + \frac{K_x}{M} + \left[\left(\frac{K_\theta}{M} - \frac{K_x}{M} \right)^2 + 4 \frac{K_x^{\,2}}{M^2} \delta^2 \right]^{\frac{1}{2}} \right\} \tag{10}$$

$$\omega_3^2 = \frac{K_y}{M} \tag{11}$$

The terms $\sqrt{K_\theta/M}$, $\sqrt{K_x/M}$ and $\sqrt{K_y/M}$ may be defined as the associated uncoupled natural frequencies of the system and will be indicated by ω'_θ, ω'_x and ω'_y, respectively. It is obvious that Eq. (11) represents motion in the y direction that is uncoupled from the x and θ motions.

The mode shapes for the remaining 2 degrees of freedom may be described in the form

$$\begin{Bmatrix} x \\ r\theta \end{Bmatrix} = \begin{Bmatrix} 1 \\ \psi \end{Bmatrix} \tag{12}$$

where parameter ψ, given by

$$\psi = \frac{2\delta}{1 - \left(\frac{\omega'_\theta}{\omega'_x}\right)^2 \pm \left\{\left[\left(\frac{\omega'_\theta}{\omega'_x}\right)^2 - 1\right]^2 + 4\delta^2\right\}^{\frac{1}{2}}} \tag{13}$$

may be defined as the degree of modal coupling.

It can be seen that if eccentricity is zero, no coupling exists, and upon the introduction of an eccentricity, coupling is created. The coupling is directly proportional to eccentricity and inversely proportional to the separation of the uncoupled natural frequencies. The expression for ψ is dimensionless and represents the additional displacement at distance r from the centre of mass that results from rotational motion when the structure oscillates in one of its natural modes.

The effect of frequency separation is particularly important when values are small, as noted in the discussion of the earlier studies. Referring to Fig. 2 where the degree of coupling is plotted versus frequency ratio, it can be seen that severe coupling occurs when the ratio ω'_θ/ω'_x approaches the value of 1. In the National Building Code of Canada (12) it is implied that a frequency separation greater than 20% is not accompanied by excessive increase in response as a result of modal coupling. With regard to Fig. 2, this value would appear reasonable for small eccentricities, i.e. $\delta < 0.01$-0.05; however, for larger values of eccentricity a separation of up to 50% may be necessary.

Effect of Building Configuration

The influence of frequency or period coincidence on modal coupling having been demonstrated, it is of interest to examine those aspects of a structure which give rise to this condition. Three basic structural parameters are involved: (1) geometry of the building in plan; (2) distribution of mass; and (3) distribution of stiffness as determined by the locations of the lateral load-resisting elements.

Although the mass and stiffness distributions can relate directly to the geometric plan of a structure, for uniform distribution of mass (the case considered in this study), the stiffness distribution and geometric layout become the principal parameters. The effect of these parameters on the frequency ratios is examined below.

Rectangular plan--For a rectangular geometry exhibiting two-fold symmetry, two limiting cases for the distribution of lateral stiffness are shown in Fig. 3.

For the uniformly distributed stiffness layout of Fig. 3(a), the rigidities in the x and y directions are

$$K_x = \int k_x dA = abk_x \tag{14}$$

$$K_y = \int k_y dA = abk_y \tag{15}$$

where K_x, K_y represent the total stiffnesses and k_x, k_y denote the unit stiffnesses in the x and y directions, respectively. Torsional rigidity is determined by

$$K_\theta = \int (y^2 k_x + x^2 k_y)\ dA \tag{16}$$

which yields

$$K_\theta = \frac{ab}{12}(b^2 k_x + a^2 k_y) \tag{17}$$

Similarly, the corresponding polar moment of inertia is obtained from

$$J = \int \rho (x^2 + y^2)\ dA \tag{18}$$

and becomes

$$J = \frac{M}{12}(a^2 + b^2) \tag{19}$$

where ρ = mass per unit area. This leads to the expression for the ratio of torsional to translational frequencies

$$\frac{\omega'_\theta}{\omega'_x} = \left[\frac{b^2 k_x + a^2 k_y}{k_x (a^2 + b^2)}\right]^{1/2} \tag{20}$$

for the x direction, whereas for the y direction* it is simply

$$\frac{\omega'_\theta}{\omega'_y} = \frac{\omega'_\theta}{\omega_x'} (k_x/k_y)^{1/2} \tag{21}$$

Referring to Fig. 4, it can be seen that for stiffness ratio $k_x/k_y = 1$ all values for aspect ratio b/a yield a frequency ratio equal to 1. For stiffness ratios different from 1, the curves tend to unity

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* The plotted curves of Figs. 4, 5, 10 and 11 can be used directly for ω'_θ/ω'_x also, by reading the indicated aspect and stiffness ratios as their corresponding inverse.

with increasing aspect ratio. Recalling Fig. 2, strong coupling is indicated when the frequency ratio approaches unity, thus for this type of plan strong coupling can be expected for any magnitude of eccentricity. For stiffness ratios different from 1, it would appear that only for a small range in aspect ratio will there be a frequency separation (i.e., $|\omega'_\theta-\omega'_x|$ or $|\omega'_\theta-\omega'_y|$) that exceeds 20%. Therefore, in general, a design incorporating a rectangular building plan and uniform distribution of lateral resistance can be expected to result in strong modal coupling.

For the plan shown in Fig. 3(b), the lateral resistance is distributed uniformly along the perimeter. The total stiffnesses for this case are

$$K_x = \int k_x dx = 2ak_x \tag{22}$$

$$K_y = \int k_y dy = 2bk_y \tag{23}$$

for translation, whereas for rotation the stiffness is

$$K_\theta = \frac{ab^2k_x + ba^2k_y}{2} \tag{24}$$

For these stiffnesses the corresponding frequency ratios are

$$\frac{\omega'_\theta}{\omega'_x} = \left[\frac{3b(bk_x+ak_y)}{k_x(a^2+b^2)}\right]^{\frac{1}{2}} \tag{25}$$

$$\frac{\omega'_\theta}{\omega'_y} = \left[\frac{3a(bk_x+ak_y)}{k_y(a^2+b^2)}\right]^{\frac{1}{2}} \tag{26}$$

The effects of aspect and stiffness ratios on the separation of frequencies are shown in Fig. 5. Over a wide range of aspect ratios, frequencies are well separated except for particular combinations of values for stiffness and aspect ratios. Strong modal coupling will therefore not generally be expected in buildings where the lateral bracing elements are located on the perimeter of the building.

<u>T-shaped plan</u>--Two examples of possible T-shaped building plan layouts with one-fold symmetry are shown in Fig. 6.

For the case of Fig. 6(a), with uniformly distributed lateral resistance, the centres of mass and resistance fall on the axis of symmetry. The following expressions for mass, polar moment of moment of inertia, and stiffnesses apply to this case:

$$M = 4\rho ab \tag{27}$$

$$J = (2.33b^2 + 1.083a^2)\rho ab \tag{28}$$

$$K_x = 4abk_x \tag{29}$$

$$K_y = 4abk_y \tag{30}$$

$$K_\theta = ab(2.333b^2k_y + 1.083a^2k_x) \tag{31}$$

With these properties, the frequency ratio for the x direction of the corresponding uncoupled system becomes

$$\frac{\omega'_\theta}{\omega'_x} = \left[\frac{1.083\ a^2k_x + 2.333b^2k_y}{k_x(1.083a^2 + 2.333b^2)}\right]^{\frac{1}{2}} \tag{32}$$

while the ratio for the y direction is given by Eq.(21).

The curves of Fig. 7 demonstrate the effects of stiffness and aspect ratios on the frequency separation to be expected for designs employing such uniform distributions of mass and lateral resistance. It is noted that the anticipated coupling action is similar to that noted for the corresponding rectangular case with uniformly distributed properties (Fig.4). This scheme is therefore likely to lead to strong coupling over a wide range of stiffness and aspect ratios.

Similarly, for the T-shaped plan layout with lateral resistance distributed along the perimeter of the building as shown in Fig. 6(b), the total stiffnesses in the principal directions are

$$K_x = 2\ \bar{k}_X \tag{33}$$

$$K_y = 2\ \bar{k}_Y \tag{34}$$

$$K_\theta = 1.125\ a^2\bar{k}_X + 4.5\ b^2\bar{k}_y \tag{35}$$

where $\bar{k}_X$ and $\bar{k}_Y$ represent total lateral stiffnesses along individual boundary lines. With the expression for mass and polar moment of inertia given by Eqs. (27) and (31), the resulting frequency ratios are given by

$$\frac{\omega'_\theta}{\omega'_x} = \sqrt{2}\left[\frac{1.125\ a^2\bar{k}_x + 4.5\ b^2\ \bar{k}_y}{\bar{k}_x(1.033a^2 + 2.333b^2)}\right]^{\frac{1}{2}} \tag{36}$$

for the x direction, and by Eq.(21) for the y direction.

Figure 8 shows the frequency separation to be expected for this case. Comparison with Fig. 5 reveals that the level of modal coupling will in general be similar to that of a rectangular plan and boundary distributed stiffness. Since the distribution of the stiffness along the boundary of the plan was selected somewhat arbitrarily, additional cases should be investigated. However, based on the data presented it is evident that a T-shaped building lends itself to strong modal coupling under conditions similar to those required for strong coupling in rectangular buildings, namely when mass and lateral resistance are distributed in approximately uniform fashion over the plan area of the building.

<u>L-Shaped plan</u>--The last building configuration considered is the

asymmetrical L-shaped plan shown in Fig, 9.

For the uniform structure of Fig. 9(a), the mass and stiffness properties are

$$M = 3\rho ab \quad (37)$$

$$J = 0.9166\rho ab\ (a^2 + b^2) \quad (38)$$

$$K_x = 3abk_x \quad (39)$$

$$K_y = 3abk_y \quad (40)$$

$$K_\theta = 0.9166ab(b^2k_x + a^2k_y) \quad (41)$$

These give the uncoupled frequency ratio for the x direction as

$$\frac{\omega'_\theta}{\omega'_x} = \left[\frac{a^2k_y + b^2\ k_x}{k_x(a^2 + b^2)}\right]^{\frac{1}{2}} \quad (42)$$

whereas for the y direction Eq. (21) applies. Strong modal coupling is to be expected for this case, as indicated by Fig. 10. However, since x and y are not principal axes, motions in these directions are always coupled and seismic response involves the effect of potentially large eccentricities in additon to the possible magnification occurring as a result of the closeness between the purely translational and rotational frequencies.

Considering the L-shaped building with peripheral stiffness arranged as shown in Fig. 9(b), the stiffnesses become

$$K_x = 2\ \bar{k}_x \quad (43)$$

$$K_y = 2\ \bar{k}_y \quad (44)$$

$$K_\theta = 2.055\ (a^2\bar{k}_y + b^2\ \bar{k}_x) \quad (45)$$

for which the corresponding ratios of associated uncoupled frequencies are give by

$$\frac{\omega'_\theta}{\omega'_x} = 1.83\left[\frac{(a^2\ \bar{k}_y + b^2\ \bar{k}_x)}{\bar{k}_x(a^2 + b^2)}\right]^{\frac{1}{2}} \quad (46)$$

and Eq. (21), for the x and y directions respectively. Referring to the curves of Fig. 11, it is clear that frequency coincidence is not expected for this case. Modal coupling for an L-shaped structure where lateral resistance is located in the exterior walls will therefore be due primarily to the effect of eccentricity between the centres of mass and stiffness and not as a result of closeness of the torsional and translational frequencies.

EXAMPLE OF MULTI-STOREY BEHAVIOUR

General Description

The simplified 3 degree of freedom structures discussed above can provide a useful means whereby the potential occurrence of severe modal coupling in the earthquake response of a structure may be identified. It is a matter of practical interest to isolate the effect of modal coupling in the seismic response of multi-storey buildings. Of particular importance are nominally symmetrical structures which are usually designed according to the code static method and an accidental eccentricity of 5% of the appropriate plan dimension. Closeness between translational and torsional periods is not included in this procedure. (The problem posed by period separation, i.e. degree of period coincidence, is considered in Commentary K where for dynamic analysis and period separation less than 20% torsional moments are to be doubled for uncoupled analysis.) Thus, the example presented herein examines the effect of period coincidence on the seismic response of a nominally symmetric three-dimensional 4-storey frame building.

Structure--The structure selected for this example, shown in Fig. 12, represents a three-dimensional version of the planar frame studied by Berg (13). The structural system retains original member stiffnesses in the plane of each frame, whereas the out-of-frame stiffness of each column corresponds to the in-plane stiffness of the intersecting perpendicular frames, thus rendering all frames in the two principal directions identical. The structure is nominally symmetrical in plan about the x and y axes.

A nominal eccentricity of 4% (i.e., δ = 0.10) is created by modifying the stiffness of exterior frames parallel to the direction of excitation. The centre of mass remains at the centre of plan. A common modification of the stiffnesses of the orthogonal frames produces the desired values of the ratio of torsional to translational periods of the original symmetrical building. Table 1 shows the relationships between the associated symmetrical (uncoupled) and the actual (coupled) ratios of torsional to translational periods. It is worth noting that a period separation greater than 20% yields nearly identical values; thus, either coupled or uncoupled periods provide the measure for degree of period coincidence in this region.

Excitation--Three statistically equal artificial earthquake acceleration records, generated using the program PSEQSN (14), were employed. The general characteristics of these are: (a) peak acceleration of 10% of gravity, (b) negligible build-up and decay times of 0.01 secs., and (c) a duration of 10 secs. Values for the damping ratio and period of the generating filter were chosen as 0.6 and 0.4 secs., respectively, to simulate motion characteristic for reasonably firm ground. Figure 13 shows a typical acceleration response spectrum for 2% damping, the latter being the value assumed for the structure.

Method of analysis--The elastic dynamic analysis was performed using the computer program TABS (15). Both time history as well as modal spectrum analysis are involved, and response is averaged for the three artificial earthquakes.

Effect of Period Separation on Seismic Response

Frame base shears--Figures 14 and 15 document the effect of the closeness between torsional and translational periods on the base shears of individual frames. It is noted that uncoupled period ratio T'_θ/T'_x affects also the total base shear of the structure as represented by the curve for average base shear. The gradual increase with decreasing T'_θ/T'_x is of particular interest; it indicates that the total base shear decreases with increasing degree of coupling*. Over the range 0-50% for period separation, total base shear increases by approximately 15%, which is in general agreement with behaviour predicted for a simple structure (9).

The response presented in Fig. 14 indicates a significant magnification of exterior frame base shears at, or near, values for period ratio of 1. It is seen that the effect of coupling is particularly pronounced only when the torsional to translational period ratio falls in the range 0.8-1.0. Increase above average frame shear ranges from 20% for frame 5 at $T'_\theta/T'_x = 0.9$ to about 40% at $T'_\theta/T'_x = 1.0$ for frame 8. As is expected, interior frames 6 and 7 experience similar but less severe changes, as seen from Fig. 15.

Modal analysis--Efficient dynamic analysis for elastic multi-storey structures is achieved by the modal spectrum technique. Total response is usually obtained by combining individual mode responses in the root-mean-square (RMS) manner. The validity of this approach has been questioned (3,4) when coupled torsional-translational response is involved. The RMS approach relies on the modes having significantly different periods, leading to different times for maximum responses. However, when the condition of near period coincidence is encountered maximum response of the two modes may occur at approximately the same time, and the resulting total response can be expected to be close to the sum of the maximum absolute values (3,4).

Figures 16 and 17 present comparisons, with respect to coincidence of the uncoupled or symmetrical periods, between the two rules for estimating maximum response using time history analysis as the actual response. While Fig. 16 represents the behaviour of the 4-storey building with uncoupled translational fundamental period $T'_x = 0.4$ secs., Fig. 17 is for the same structure modified to model a flexible building with period $T'_x = 2.0$ secs.

For the flexible structure (Fig. 17) the RMS spectrum analysis provides acceptable estimates of maximum response. However, the rigid structure (Fig. 16) is sensitive to coupling through period ratio, with

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* Additional response data not reported herein (16) indicate that increasing the eccentricity between centres of mass and stiffness also decreases the total base shear.

the result that the RMS approach yields good estimates only when the period separation exceeds 20-30%. For closer periods, the RMS and the sum of absolute modal contributions both exhibit considerable error. Averaging of the values obtained for the two approaches would seem to yield proper estimates of maximum response in this range. What this difference in behavior for the two structures with periods of 0.4 and 2.0 secs. implies is that the effect of coupling between translational and torsional response is considerably more severe for relatively stiff buildings than for buildings that are flexible. This is in general agreement with a similar observation noted in a study (5) of response to torsional excitation.

SUMMARY AND CONCLUSIONS

Simplified 3 degree of freedom structures, both of symmetrical and irregular plan geometry, have been studied from the viewpoint of providing guidelines in identifying situations that lead to severe coupling effects during earthquake excitation. The following conclusions may be drawn regardless of the plan configuration of the building:

(1) The degree of coupling, and hence maximum response during excitation, depends directly on the eccentricity between centres of mass and lateral resistance and inversely on the separation of the torsional and translational frequencies of the corresponding uncoupled structure.

(2) When lateral resistance is arranged uniformly over the plan of the building, as is the case normally encountered for column and flat slab construction for example, torsional and translational frequencies tend to have similar values and, consequently, the effects of modal coupling can be expected to be severe.

(3) When lateral resistance is located on the perimeter of the building, torsional frequencies are larger than the translational frequencies and the dynamic effects of coupling will be less severe.

The example of the 4-storey frame building having nominal eccentricity of 4% and subjected to three earthquake records indicates the following additional conclusions when the degree of coupling is varied through the frequency ratio:

(4) The total base shear decreases as the degree of coupling is increased.

(5) The torsional effect becomes maximum for the critical perimeter elements when period separation for the associated uncoupled system approaches zero. However, the effect of coupling is not significant when the period separation exceeds 20%, and the code guidelines therefore appear adequate in this regard.

(6) If modal spectrum analysis is employed to predict maximum seismic response, the RMS approach appears suitable for flexible structures, whereas for short-period structures this approach underestimates maximum response considerably and the average of RMS and sum of maximum absolute modal contributions seems more appropriate. For situations when the separation between the lowest torsional and translational frequencies approaches 0, in terms of either the coupled or corresponding uncoupled

system, a time history dynamic analysis should be performed.

(7)The preceding observation implies that in general short, relatively stiff, buildings are more susceptible to severe effects of modal coupling than buildings that are tall and relatively flexible.

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TABLE 1: Coupled and Uncoupled Period Ratios

Coupled Periods (secs)			Uncoupled Period Ratio
T_x	T_θ	T_θ/T_x	T'_θ/T_x'
0.42	0.38	0.91	1.0
0.41	0.37	0.90	0.95
0.41	0.35	0.87	0.90
0.40	0.34	0.83	0.85
0.40	0.32	0.79	0.80
0.40	0.28	0.69	0.70
0.40	0.24	0.60	0.60
0.40	0.20	0.50	0.50

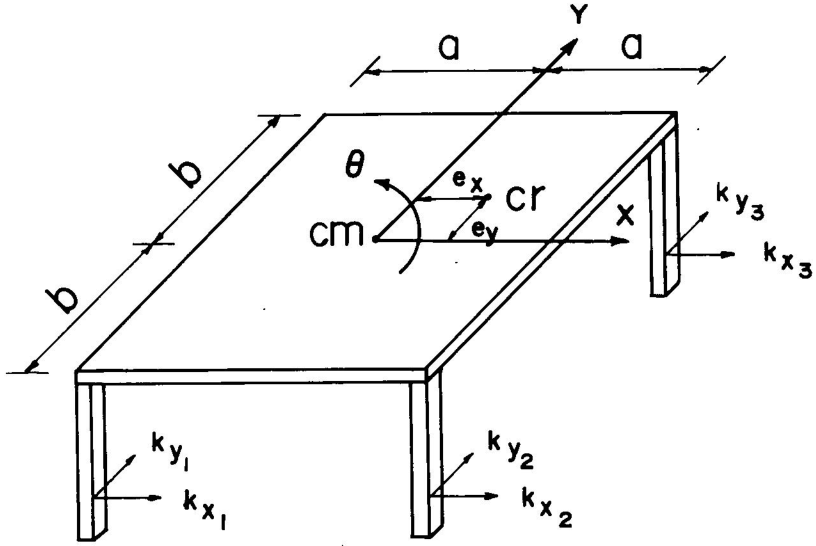


Fig. 1 Idealized One-Storey Structural Model

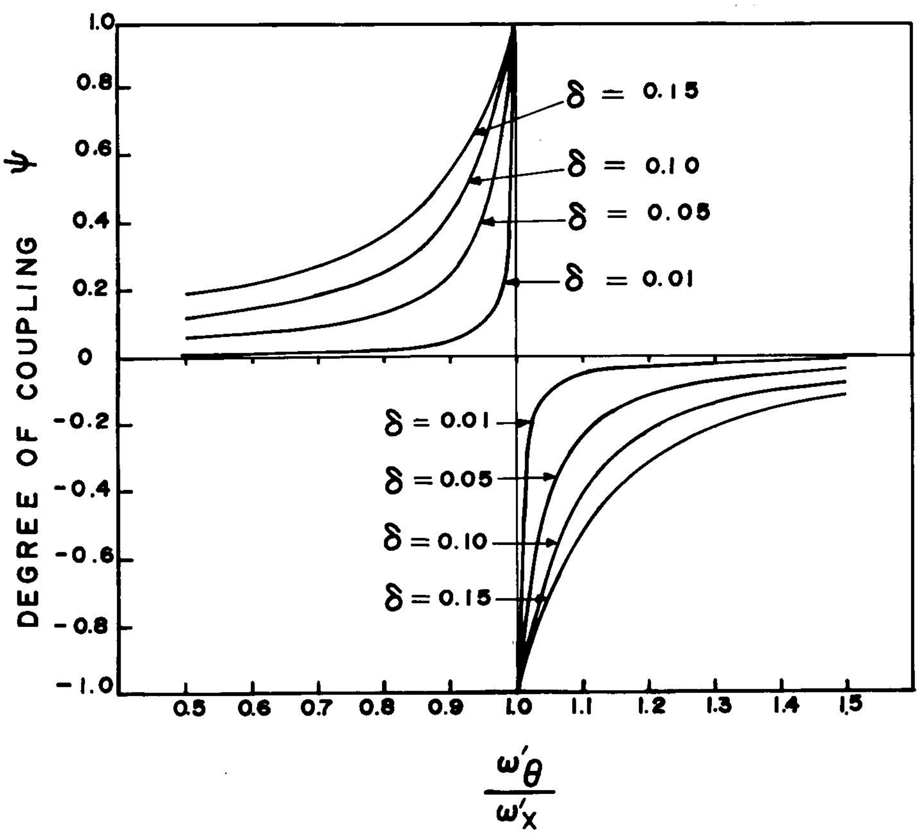


Fig. 2 Influence of Frequency Ratio on Degree of Coupling

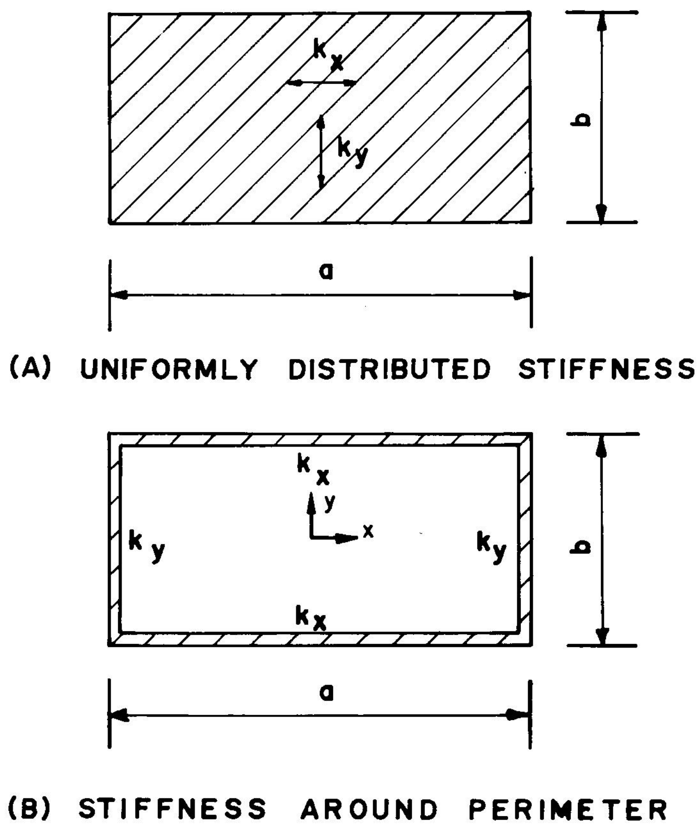


Fig. 3 Rectangular Buildings

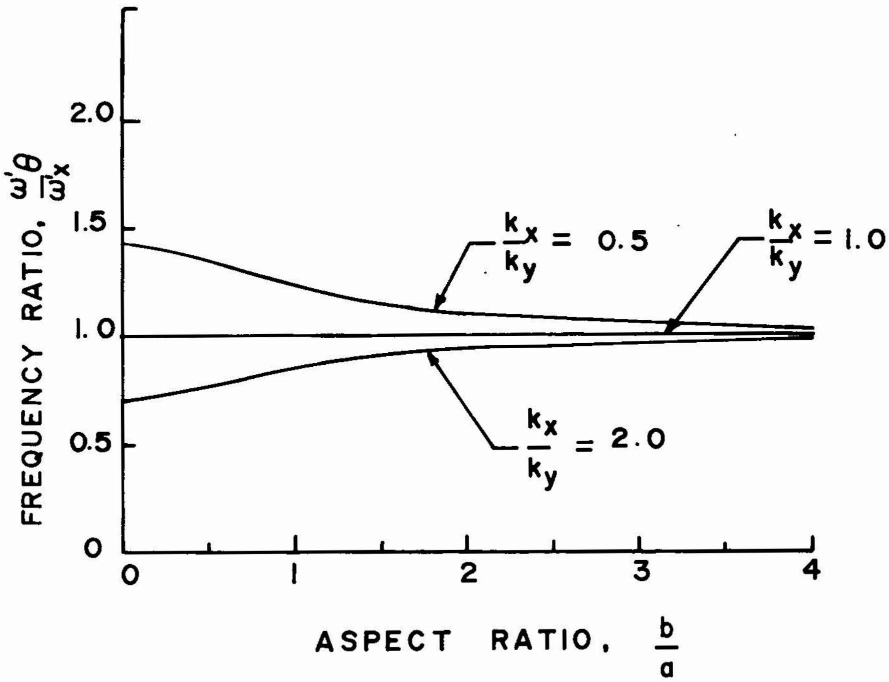


Fig. 4 Frequency Ratio for Rectangular Buildings with Uniformly Distributed Resistance

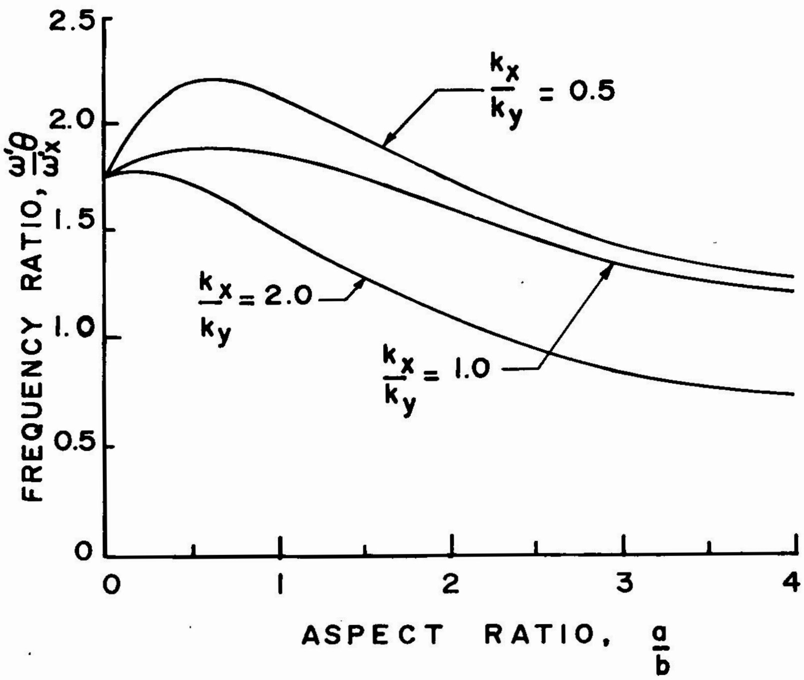


Fig. 5 Frequency Ratio for Rectangular Buildings with Peripheral Resistance

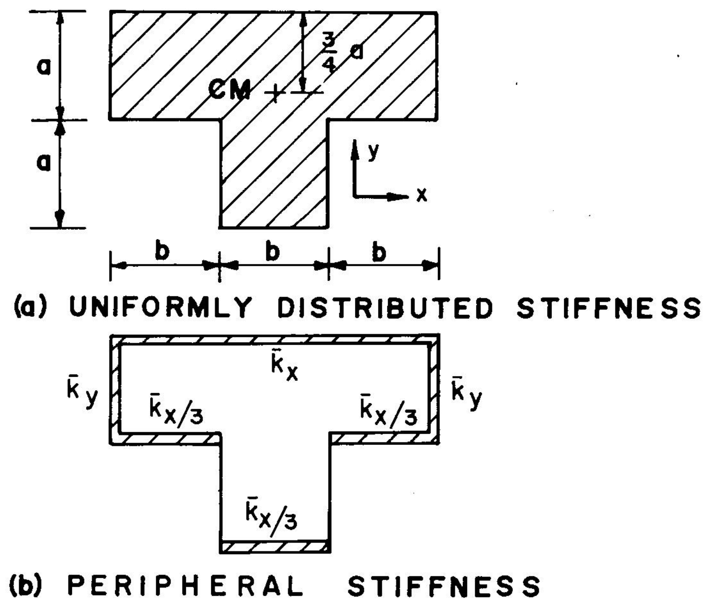


Fig. 6 T-Shaped Buildings

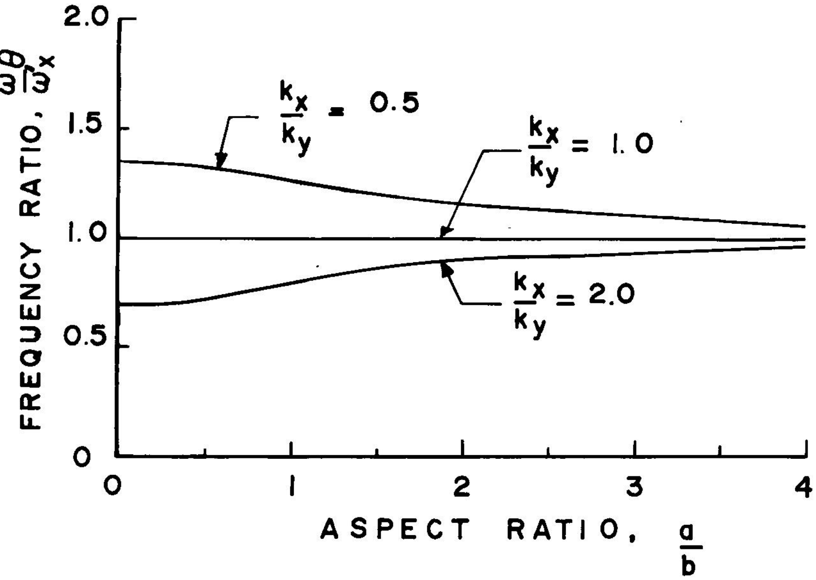


Fig. 7 Frequency Ratio for T-Shaped Buildings with Uniformly Distributed Resistance

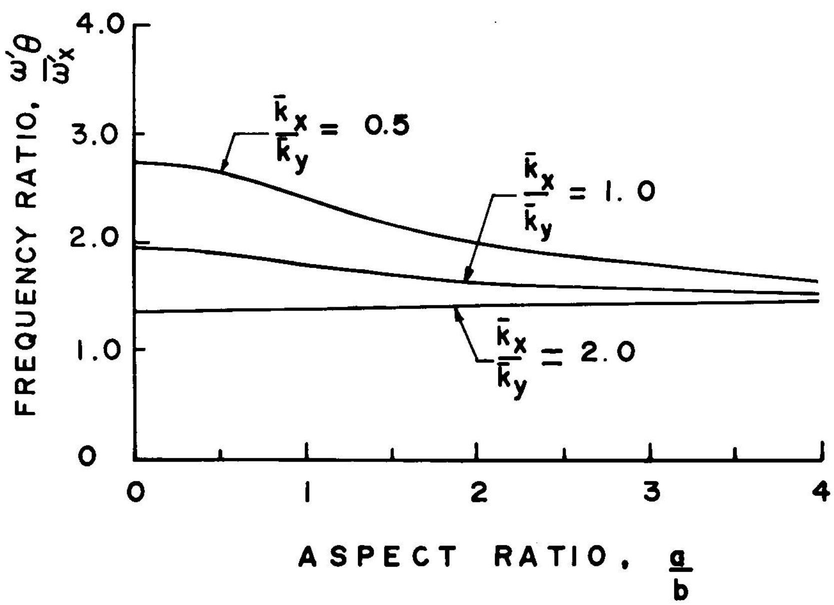
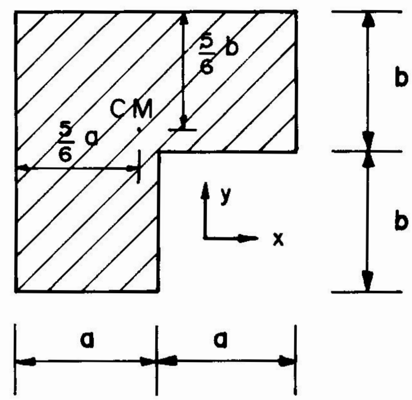
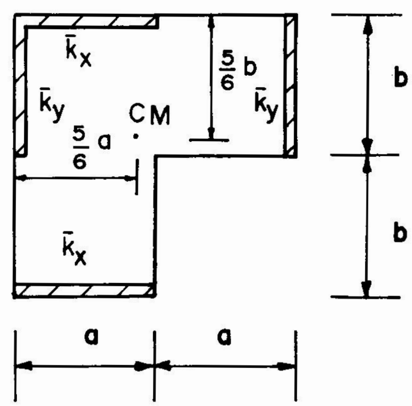


Fig. 8 Frequency Ratio for T-Shaped Buildings with Peripheral Resistance



(a) UNIFORMLY DISTRIBUTED STIFFNESS



(b) PERIPHERAL STIFFNESS

Fig. 9 L-Shaped Buildings

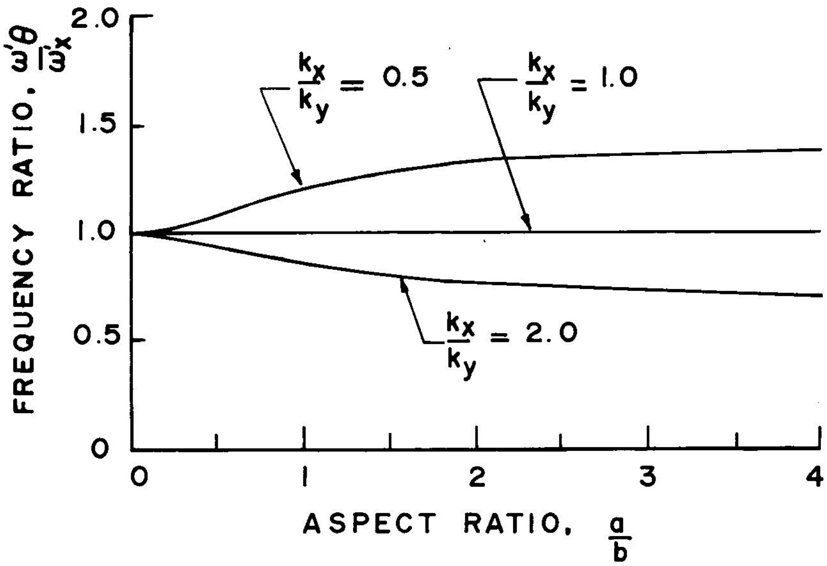


Fig. 10 Frequency Ratio for L-Shaped Buildings

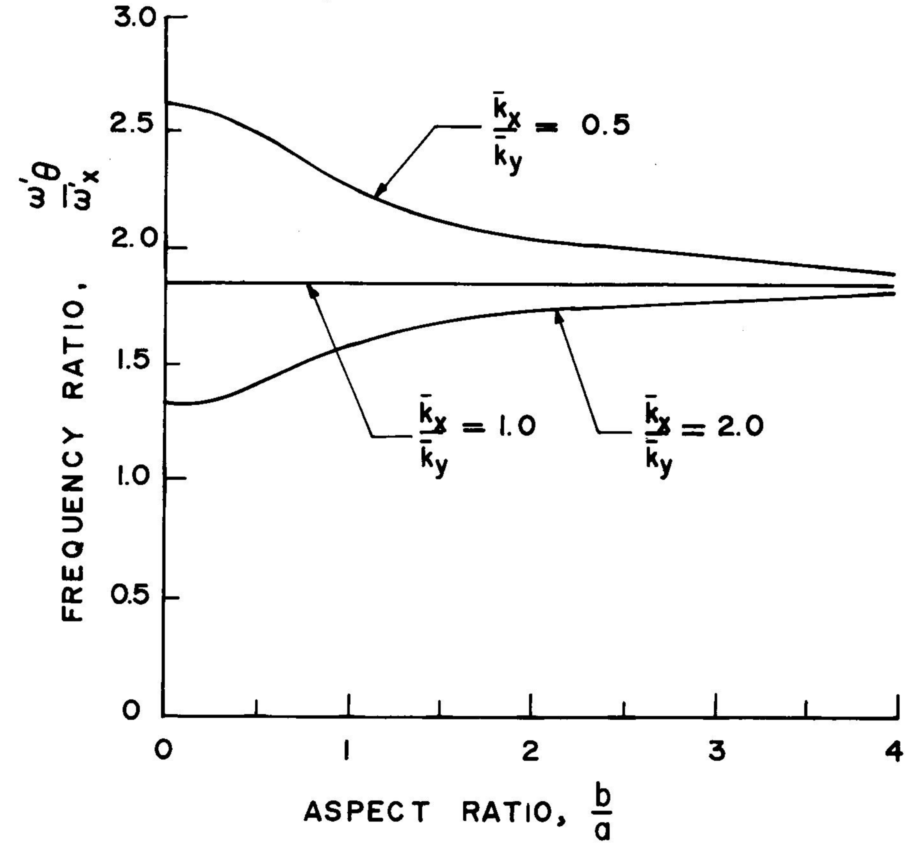


Fig. 11 Frequency Ratio for L-Shaped Buildings with Peripheral Stiffness

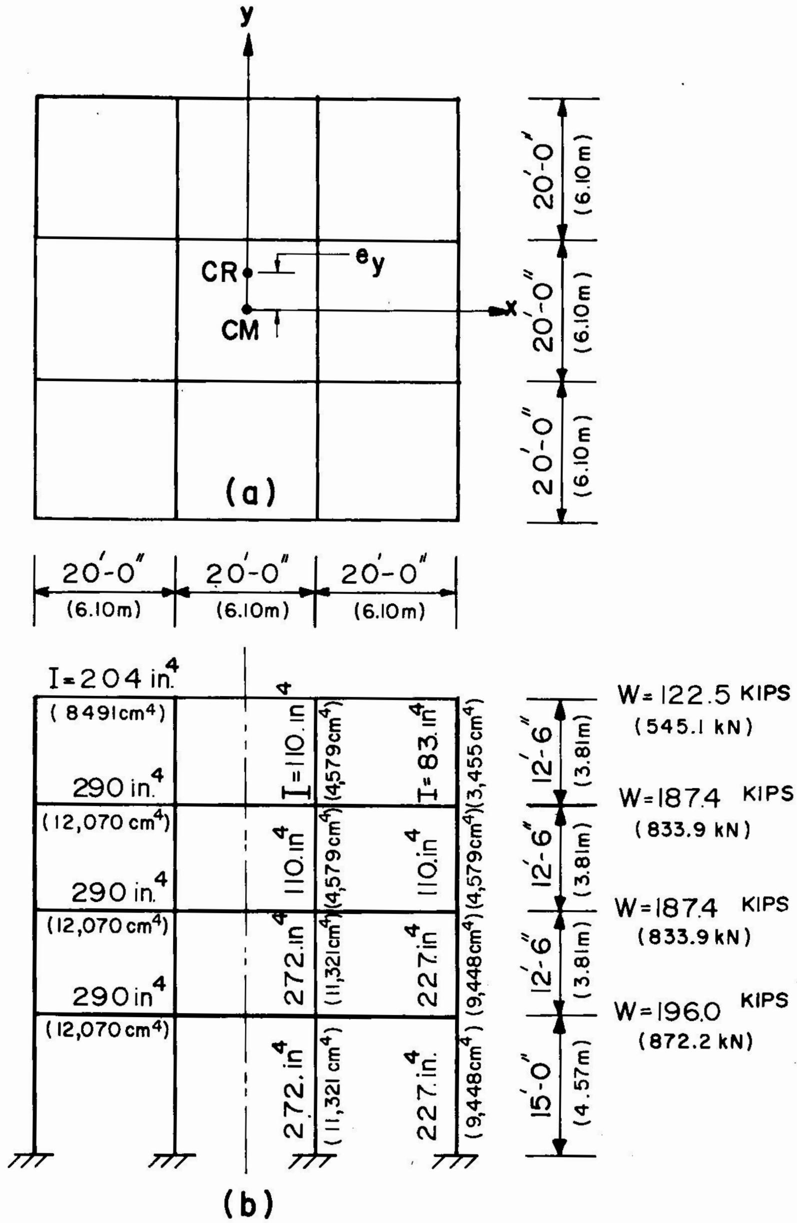


Fig. 12 Four-Storey Frame Building: (a) Plan; (b) Elevation of Typical Frame

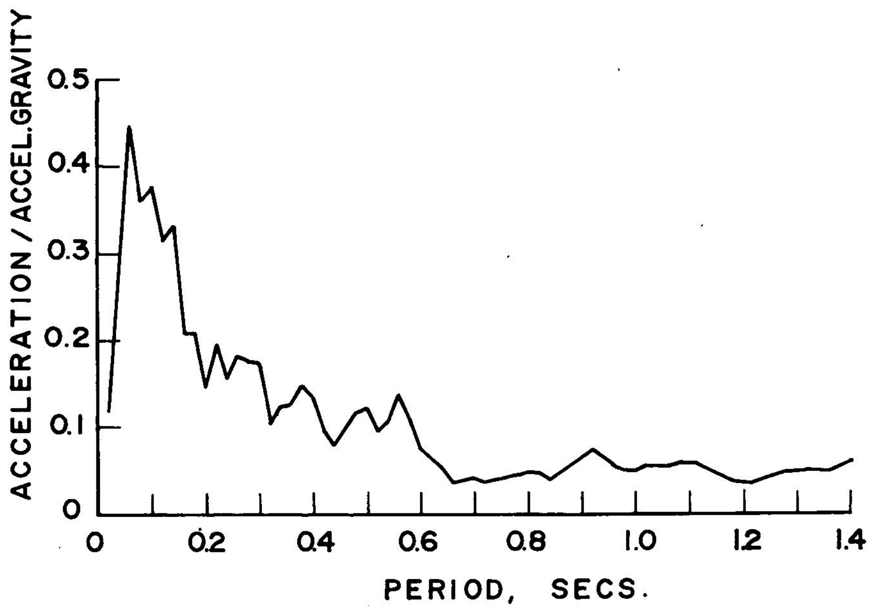


Fig. 13 Acceleration Response Spectrum No. 2 for 2% Damping

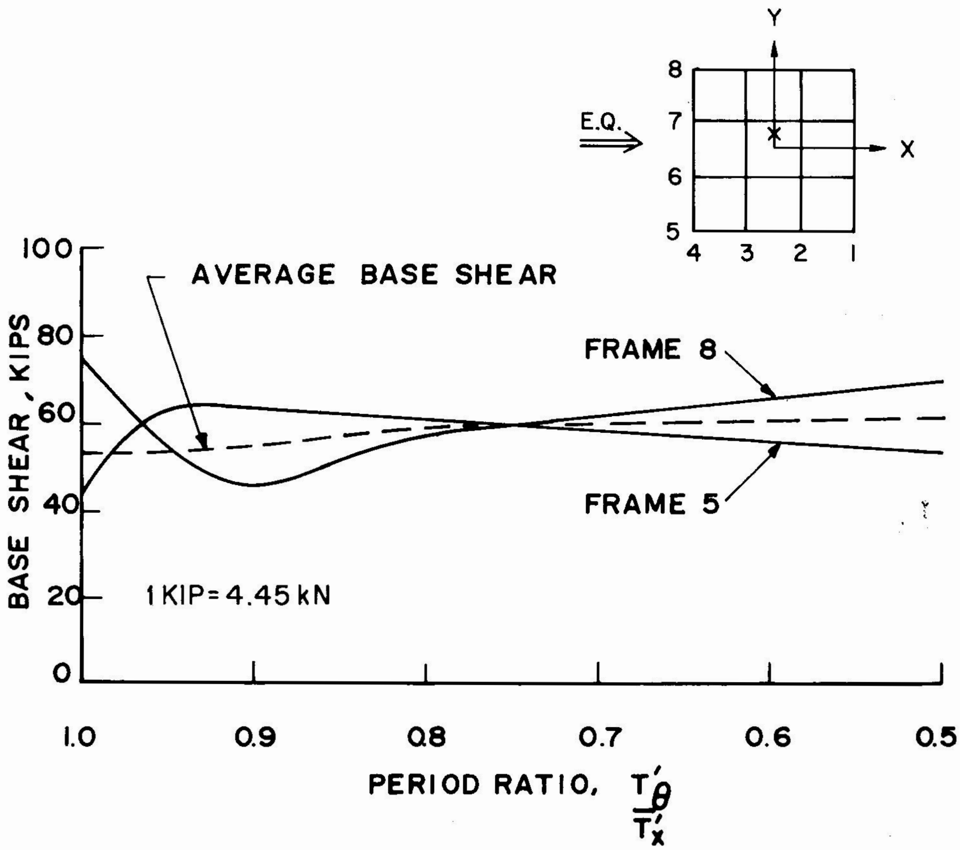


Fig. 14 Base Shears of Exterior Frames (Averaged Time History Analyses)

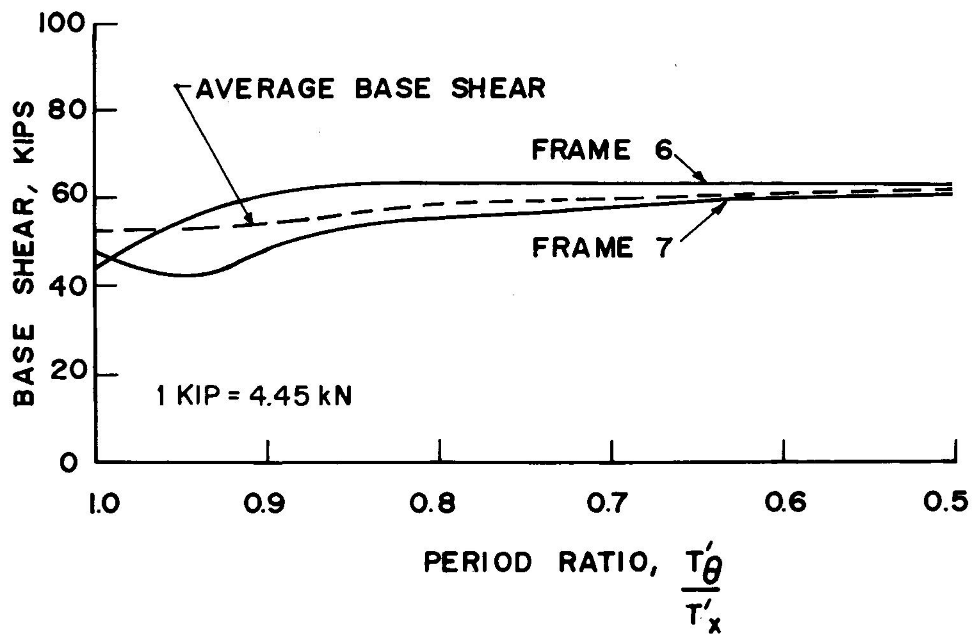


Fig. 15 Base Shears of Interior Frames (Averaged Time History Analyses)

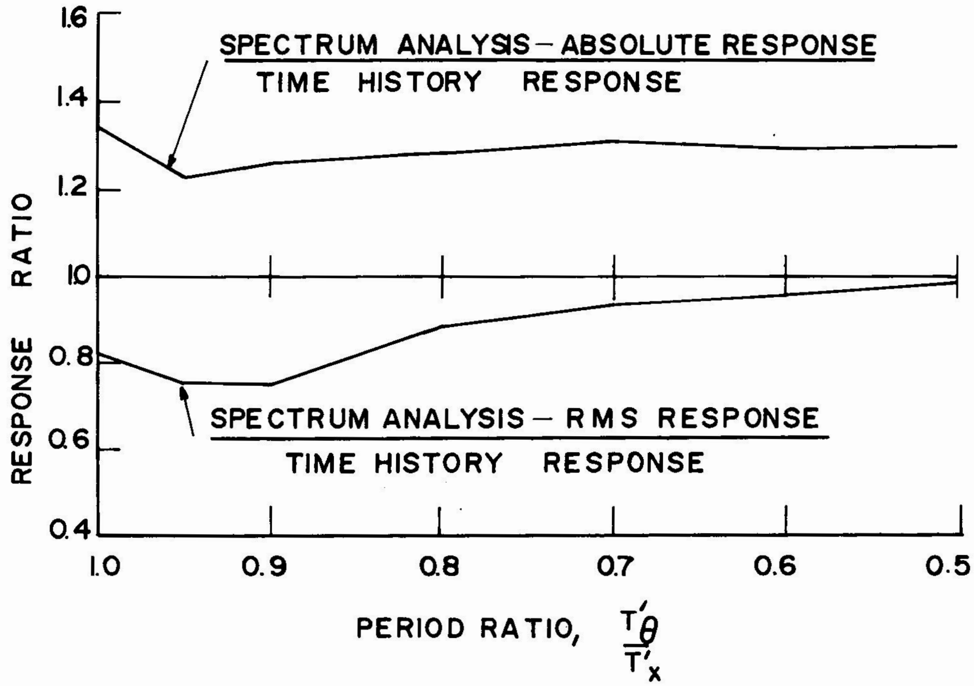


Fig. 16 Comparison of Methods of Analysis for Rigid Building (Average of Base Shears; Frames 5-8)

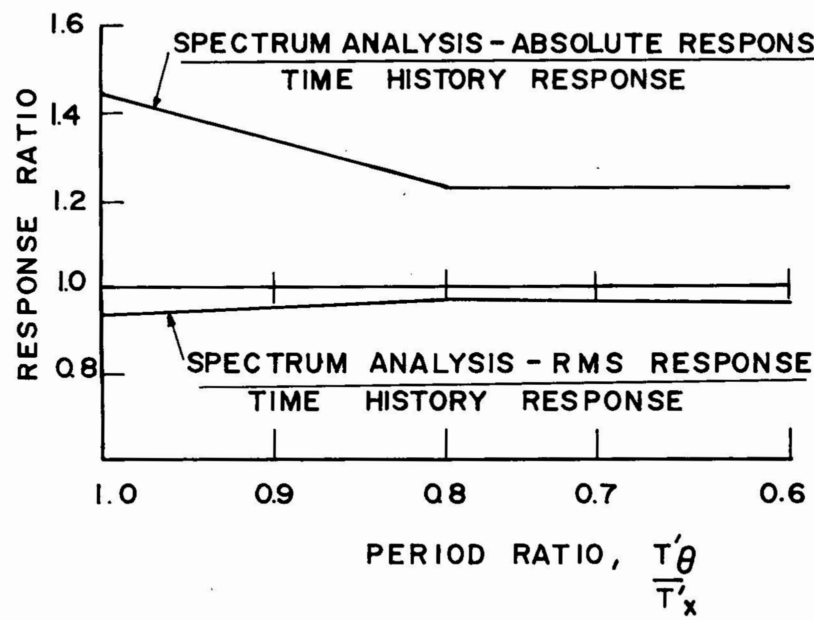


Fig. 17 Comparison of Methods of Analysis for Flexible Building (Average of Base Shears; Frames 5-8)